range (4.9%) associated with the addition of a ceramic coating (T_w = 4000°F, ϕ_T = 0.7) to a heat exchanger operating at 1600°F is noted in Fig. 2. Even though the addition of a ceramic coating permits operation at the minimum equivalence ratio considered in this study, ϕ_T = 0.7, the range increase (4.9%) is less than for the uncoated heat exchanger (7.7% at T_w = 2200°F, ϕ_T = 0.7) because of the large increase in engine weight associated with the addition of a ceramic coating. Furthermore, since the coating must be sized to accommodate the increased heating rates that would be experienced during a turning maneuver or off-design conditions, it will operate less effectively than previously indicated because of its lower effective wall temperature at cruise conditions. The designer must therefore examine the use of thermal coatings very carefully to insure that the gain in performance justifies added development and maintenance costs.

Conclusions

The results of this study indicate that, with a regenerative system using superalloy heat exchangers and the hydrogen fuel as coolant, Mach 12 cruise at equivalence ratios less than one can be achieved. The increase in range and/or decrease in operating costs possible with T. D. nickel-chrome heat exchangers suggest their development. The development of refractory metal heat exchangers or ceramic coatings appears less urgent. Present and planned ground test facilities are considered adequate for initial determination of heat-transfer characteristics and heat exchanger development. However, to insure successful operation of the complete engine cooling system, flight test of a representative engine module is required.

Airfoil Pressure Distributions in Low-Speed Stall

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Good accuracy in the calculation of pressure distributions upon arbitrary, two-dimensional, unstalled airfoils satisfying a Kutta condition has been achieved.[1] Much less attention has been given to the estimation of pressures upon stalled airfoils, i.e., wing sections from which the flow has separated completely to form a broad wake that dominates the flowfield. In order to analyze the dynamics of stall recovery, it is necessary to have such pressure distributions in hand.

It has been observed that the near wakes of stalled airfoils tend to support only mild pressure gradients, in both the streamwise and lateral directions.[2] Large pressure gradients cannot exist unless there are substantial velocity gradients, and the separated region is characterized by slow and even recirculating velocities. These ideas are implicit in the many hodograph treatments (such as Ref. 3) that concern the inverse problem, i.e., finding the streamline pattern for a given pressure level.

The present Note describes a computerized method for estimating the pressure distribution on a stalled airfoil with an arbitrary known or assumed wake geometry. The method employs the approximation that pressures are transmitted undiminished from the outer boundary of the wake in the direction normal to the airfoil. This work complements the treatment of Ref. 1 for airfoils without separation.

To obtain the perfect fluid (potential) flow solution about the sheath defined by the airfoil and its wake, a Schwarz-Christoffel transformation of plane stagnation flow is used. The transformation is attractive in its generality, for it relates a straight line to any shape that can be approximated by a closed or open polygon.[4] Plane stagnation flow is appropriate because it features a streamline that splits at a stagnation point and travels to infinity along two separate paths, as does

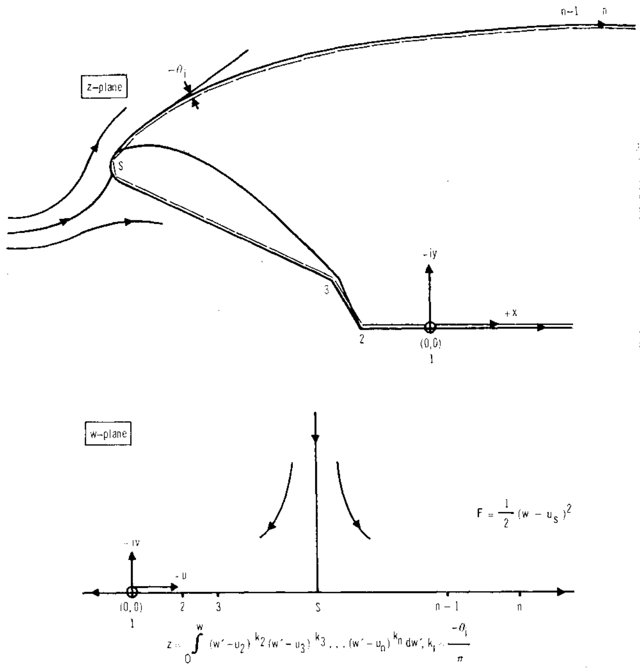

Fig. 1 The Schwarz-Christoffel transformation.

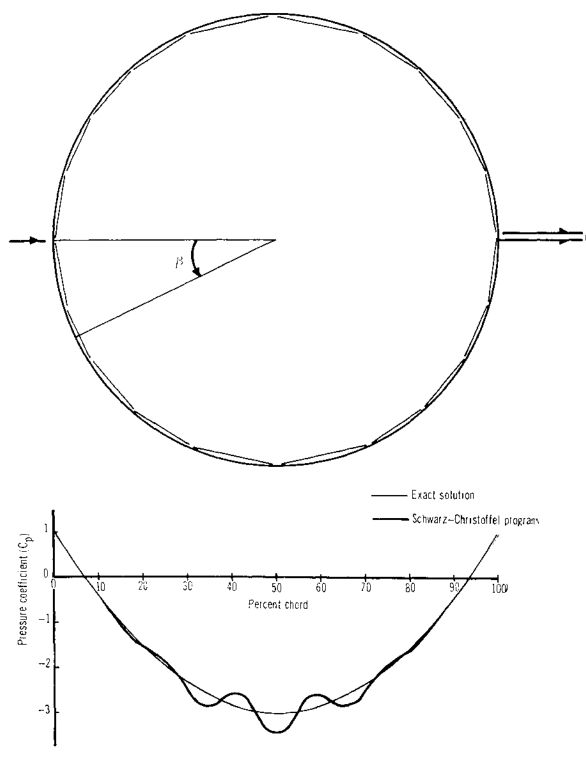

Fig. 2 Pressure distribution on a round cylinder with zero circulation.

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the bounding streamline of the airfoil/wake (see Fig. 1). The describing equations are as follows:

$$F(w) = \tfrac{1}{2}(w - u_s)^2 = \varphi + i\psi \tag{1}$$

$$z(w) = \int_0^w (w' - u_2)^{k_2}(w' - u_3)^{k_3} \dots (w' - u_n)^{k_n}\, dw' \tag{2}$$

$$k_i = \text{angle change}/\pi$$

$$F(z) = F[w(z)] \tag{3}$$

$$\bar{V}/V_\infty = dF/dz = V_x/V_\infty - iV_y/V_\infty \tag{4}$$

$$C_p = (p - p_\infty)/\tfrac{1}{2}\rho V_\infty^2 = 1 - (V/V_\infty)^2 \tag{5}$$

It is necessary to specify the forward stagnation point on the underside of the airfoil. Examination of pressure distribution data suggests the location 2% chord, for cases in which the separation is near the leading edge.

The chief difficulty in using the transformation is that the deeply embedded parameters $u_2, u_3, \dots, u_n$ in Eq. (2) must be obtained by some iterative procedure based on matching the "vertex" coordinates $z_2, z_3, \dots, z_n$. This is done through a complex arithmetic, double precision FORTRAN routine for the IBM 360, written by the author. Numerical quadrature of the complex integrand in Eq. (2), taking proper account of the (integrable) singularities, yields a z_i set for any u_i set. This calculated z_i set is compared with the inputted (z_i) geometry, and the first-order, multivariable Newton's method[4] is used to generate a set of Δu_i corrections. The process can be made to converge with arbitrary accuracy. The substance of this work, therefore, is the application of current electronic computer capabilities to an established, general mathematical method.

Each concave corner is a stagnation point and each convex corner is a point of infinite velocity, in perfect fluid theory. However, calculation of pressure distributions by Eqs. (4) and (5) shows that these singularities are important only locally and their effects may be smoothed over or ignored.

The round cylinder without circulation provides a standard test for any perfect fluid calculation method. The geometry of the infinitesimal "wake" and the location of the forward stagnation point are determined by symmetry. Figure 2 shows the result of applying the present program to the purposely crude polygonal approximation as drawn, compared with the well-known solution[5]:

$$C_{p_{cyl}} = 1 - 4\sin^2\beta \tag{6}$$

It is seen that the computer answer does agree with the standard solution on the average (and oscillates about it).

Little test data including both stalled airfoil pressure distributions and the corresponding wake geometry are found in the literature. However, Fig. 3 shows a comparison between calculated answers and pressure measurements found in Ref. 6. Agreement is no better than fair. The near wake form was taken from a sketch in that report, which was generated from two locus points of "definite turbulence" as indicated by fine threads on movable wire probes. In addition to the questionable accuracy of the wake geometry, it is specifically noted in Ref. 6 that model blockage effects caused the upper surface C_p measurements to be more negative than they would have been in free air. It is not possible at this time to weigh the imprecision of the experiment against the error incurred by the use of the idealized wake model.

Experimental measurement of at least gross wake geometry and separated airfoil pressure distributions would be very helpful. This information, together with rates of dynamic pressure recovery in the unsymmetrical near wake, is also needed for systematic study of T-tail deep stall.

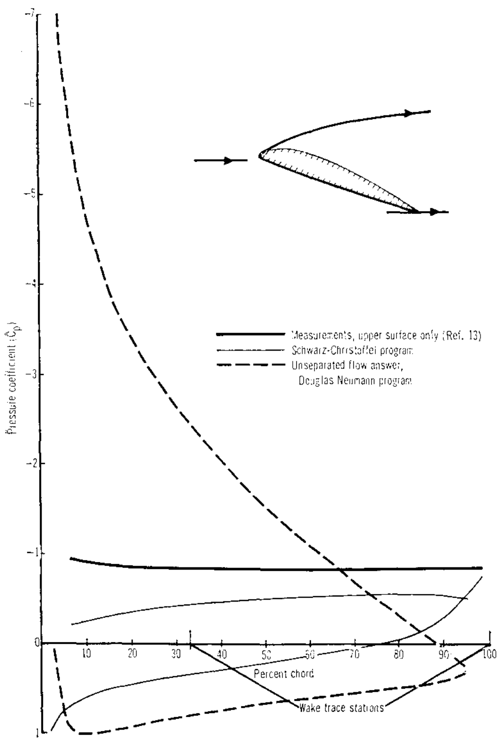

Fig. 3 Pressure distribution on English Aircrew 4 at 19° incidence.

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Optimum Climb Trajectories at Constant Lift Coefficient

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FOR many years, it has been possible to calculate aircraft minimum-time-to-climb trajectories by use of the calculus of variations. The resulting flight paths, however, typically show a continual variation in aerodynamic and control parameters from one time point to the next. Consequently, it is always very difficult, if not impossible, for a human pilot to duplicate this optimal path. Usually the best that can be done is to try to fly the trajectory in a series of separate segments.

It has now been found that if an aircraft is flown throughout the climb at a constant lift coefficient, a minimum-time trajectory can be closely approximated.

Figure 1 shows the performance of a typical fighter-type aircraft in a climb from 25,000 to 46,000 ft. The optimum

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